

Initiation_Python_Jupyter_TP2

October 11, 2021

In [2]: `random()`

```
-----
NameError                                 Traceback (most recent call last)
<ipython-input-2-347a394b3b57> in <module>
----> 1 random()

NameError: name 'random' is not defined
```

In [3]: `import numpy
print(numpy.pi)
print(numpy.e)
print(numpy.euler_gamma)`

```
3.141592653589793
2.718281828459045
0.5772156649015329
```

In [4]: `print(numpy.random.rand())
print(numpy.random.randint(1,10))`

```
0.7644154963495916
7
```

In [5]: `print(numpy.random.binomial(100,0.25,10))
print()
help(numpy.random.binomial)`

```
[28 25 19 28 21 36 20 24 31 28]
```

Help on built-in function binomial:

```
binomial(...) method of mtrand.RandomState instance
    binomial(n, p, size=None)
```

Draw samples from a binomial distribution.

Samples are drawn from a binomial distribution with specified parameters, n trials and p probability of success where n an integer ≥ 0 and p is in the interval [0,1]. (n may be input as a float, but it is truncated to an integer in use)

Parameters

n : int or array_like of ints

Parameter of the distribution, ≥ 0 . Floats are also accepted, but they will be truncated to integers.

p : float or array_like of floats

Parameter of the distribution, ≥ 0 and ≤ 1 .

size : int or tuple of ints, optional

Output shape. If the given shape is, e.g., ```(m, n, k)```, then ```m * n * k``` samples are drawn. If size is ```None``` (default), a single value is returned if ```n``` and ```p``` are both scalars.

Otherwise, ```np.broadcast(n, p).size``` samples are drawn.

Returns

out : ndarray or scalar

Drawn samples from the parameterized binomial distribution, where each sample is equal to the number of successes over the n trials.

See Also

`scipy.stats.binom` : probability density function, distribution or cumulative density function, etc.

Notes

The probability density for the binomial distribution is

$$\dots \text{math:: } P(N) = \binom{n}{N} p^N (1-p)^{n-N},$$

where `:math:`n`` is the number of trials, `:math:`p`` is the probability of success, and `:math:`N`` is the number of successes.

When estimating the standard error of a proportion in a population by using a random sample, the normal distribution works well unless the product `p*n <= 5`, where p = population proportion estimate, and n = number of samples, in which case the binomial distribution is used

instead. For example, a sample of 15 people shows 4 who are left handed, and 11 who are right handed. Then $p = 4/15 = 27\%.$ $0.27*15 = 4,$ so the binomial distribution should be used in this case.

References

- .. [1] Dalgaard, Peter, "Introductory Statistics with R", Springer-Verlag, 2002.
- .. [2] Glantz, Stanton A. "Primer of Biostatistics.", McGraw-Hill, Fifth Edition, 2002.
- .. [3] Lentner, Marvin, "Elementary Applied Statistics", Bogden and Quigley, 1972.
- .. [4] Weisstein, Eric W. "Binomial Distribution." From MathWorld--A Wolfram Web Resource.
<http://mathworld.wolfram.com/BinomialDistribution.html>
- .. [5] Wikipedia, "Binomial distribution",
https://en.wikipedia.org/wiki/Binomial_distribution

Examples

Draw samples from the distribution:

```
>>> n, p = 10, .5 # number of trials, probability of each trial
>>> s = np.random.binomial(n, p, 1000)
# result of flipping a coin 10 times, tested 1000 times.
```

A real world example. A company drills 9 wild-cat oil exploration wells, each with an estimated probability of success of 0.1. All nine wells fail. What is the probability of that happening?

Let's do 20,000 trials of the model, and count the number that generate zero positive results.

```
>>> sum(np.random.binomial(9, 0.1, 20000) == 0)/20000.
# answer = 0.38885, or 38%.
```

```
In [6]: print(numpy.cos(numpy.e**2))
      print(numpy.arccos(0.4))
      print(numpy.e**2-2*numpy.pi)
```

```
0.44835624181873357
1.1592794807274085
1.1058707917510633
```

```
In [7]: from numpy import *
import numpy.random as npr
```

```
print(pi)
print(e)
print(random.rand())
print(npr.rand())
print(rand())

3.141592653589793
2.718281828459045
0.6228003968508204
0.9028626208717166
```

NameError

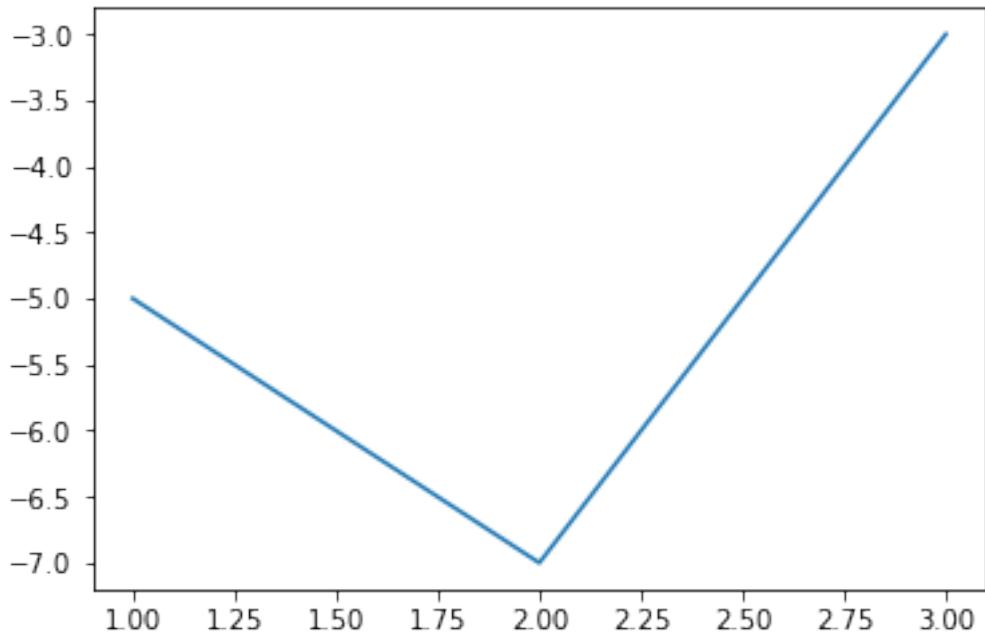
Traceback (most recent call last)

```
<ipython-input-7-44c03736a0b2> in <module>
      6 print(random.rand())
      7 print(npr.rand())
----> 8 print(rand())
```

NameError: name 'rand' is not defined

```
In [8]: import matplotlib.pyplot as plt
        from scipy import *
        from math import factorial as fact
```

```
In [9]: plt.plot([1,2,3], [-5,-7,-3])
        plt.show()
```



```
In [10]: help(numpy.random.chisquare)
```

```
Help on built-in function chisquare:
```

```
chisquare(...) method of mtrand.RandomState instance
    chisquare(df, size=None)
```

```
Draw samples from a chi-square distribution.
```

When `df` independent random variables, each with standard normal distributions (mean 0, variance 1), are squared and summed, the resulting distribution is chi-square (see Notes). This distribution is often used in hypothesis testing.

Parameters

df : float or array_like of floats

Number of degrees of freedom, should be > 0.

size : int or tuple of ints, optional

Output shape. If the given shape is, e.g., ``(m, n, k)``, then ``m * n * k`` samples are drawn. If size is ``None`` (default), a single value is returned if ``df`` is a scalar. Otherwise, ``np.array(df).size`` samples are drawn.

Returns

```
out : ndarray or scalar
    Drawn samples from the parameterized chi-square distribution.

Raises
-----
ValueError
    When `df` <= 0 or when an inappropriate `size` (e.g. ``size=-1``)
    is given.
```

Notes

```
-----  
The variable obtained by summing the squares of `df` independent,  
standard normally distributed random variables:
```

```
.. math:: Q = \sum_{i=0}^{\mathit{df}} X^2_i  
is chi-square distributed, denoted  
.. math:: Q \sim \chi^2_k.
```

The probability density function of the chi-squared distribution is

```
.. math:: p(x) = \frac{(1/2)^{k/2}}{\Gamma(k/2)} x^{k/2 - 1} e^{-x/2},
```

where `:math:`\Gamma`` is the gamma function,

```
.. math:: \Gamma(x) = \int_0^{-\infty} t^{x - 1} e^{-t} dt.
```

References

```
-----  
.. [1] NIST "Engineering Statistics Handbook"  
      https://www.itl.nist.gov/div898/handbook/eda/section3/eda3666.htm
```

Examples

```
-----  
=>> np.random.chisquare(2,4)  
array([ 1.89920014,  9.00867716,  3.13710533,  5.62318272])
```

```
In [11]: L = [2,3,5,7,11,13,17,19]  
print(L)  
print(type(L))  
print(L[0])  
print(L[3])  
print(L[-1])  
print(len(L))
```

```

print(L[:4])
print(L[4:])
print(L[1:6])

[2, 3, 5, 7, 11, 13, 17, 19]
<class 'list'>
2
7
19
8
[2, 3, 5, 7]
[11, 13, 17, 19]
[3, 5, 7, 11, 13]

```

```

In [12]: L.append(23)
          print(L)
          L.append(29)
          print(L)
          L.append("N'importe quoi")
          print(L)
          L.append([1,1,2,3,5,8,13,21])
          print(L)
          L[-1].append(34)
          print(L)

```

```

[2, 3, 5, 7, 11, 13, 17, 19, 23]
[2, 3, 5, 7, 11, 13, 17, 19, 23, 29]
[2, 3, 5, 7, 11, 13, 17, 19, 23, 29, "N'importe quoi"]
[2, 3, 5, 7, 11, 13, 17, 19, 23, 29, "N'importe quoi", [1, 1, 2, 3, 5, 8, 13, 21]]
[2, 3, 5, 7, 11, 13, 17, 19, 23, 29, "N'importe quoi", [1, 1, 2, 3, 5, 8, 13, 21, 34]]

```

```

In [13]: L.insert(0,1)
          print(L)
          L.insert(11,31)
          print(L)
          L[-1].insert(0,0)
          print(L)

```

```

[1, 2, 3, 5, 7, 11, 13, 17, 19, 23, 29, "N'importe quoi", [1, 1, 2, 3, 5, 8, 13, 21, 34]]
[1, 2, 3, 5, 7, 11, 13, 17, 19, 23, 29, 31, "N'importe quoi", [1, 1, 2, 3, 5, 8, 13, 21, 34]]
[1, 2, 3, 5, 7, 11, 13, 17, 19, 23, 29, 31, "N'importe quoi", [0, 1, 1, 2, 3, 5, 8, 13, 21, 34]]

```

```

In [14]: M = [1,2,3]
          N = [4,5,6]
          print(M,N)
          K = M+N

```

```
print(K)
print(M,N)
```

```
[1, 2, 3] [4, 5, 6]
[1, 2, 3, 4, 5, 6]
[1, 2, 3] [4, 5, 6]
```

```
In [15]: # L = []
# for k in range(100):
#     L.append(np.random.rand())
```

```
L = [np.random.rand() for k in range(100)]
print(L[:20])
L.sort()
print(L[:20])
```

```
[0.6904718987457737, 0.27602967675694545, 0.9031961023170058, 0.05292021204701536, 0.591587462
[0.005998683497628554, 0.010538130278089364, 0.02295180838868116, 0.04584621747558537, 0.05292
```

```
In [16]: L = [(i,j) for i in range(1,21) for j in range(1,21) if i-j >= 10]
print(L)
```

```
L = [np.random.rand() for k in range(100)]
M = [x for x in L if x > 1/2]
print(len(M),min(M))
```

```
[(11, 1), (12, 1), (12, 2), (13, 1), (13, 2), (13, 3), (14, 1), (14, 2), (14, 3), (14, 4), (15
47 0.5036075807762839
```

```
In [17]: n = 3628800
D = [d for d in range(1,n+1) if n%d == 0]
print(len(D),D[100])
```

```
270 700
```

```
In [76]: def nombre_de_diviseurs(n):
    c = 0
    for d in range(1,n+1):
        if n%d == 0:
            c += 1
    return c

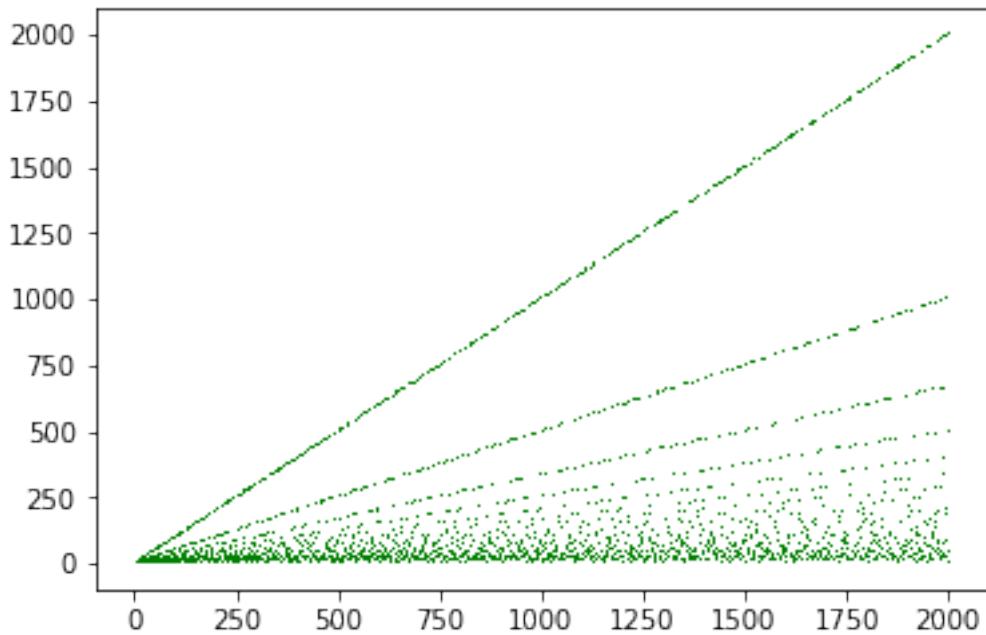
def est_premier(p):
    return nombre_de_diviseurs(p) == 2
```

```

def plus_grand_diviseur_premier(n):
    for d in range(n, 1, -1):
        if n%d == 0:
            if est_premier(d):
                return d

N = [n for n in range(2, 2001)]
P = [plus_grand_diviseur_premier(n) for n in N]
plt.plot(N, P, 'g,')
plt.show()

```



```

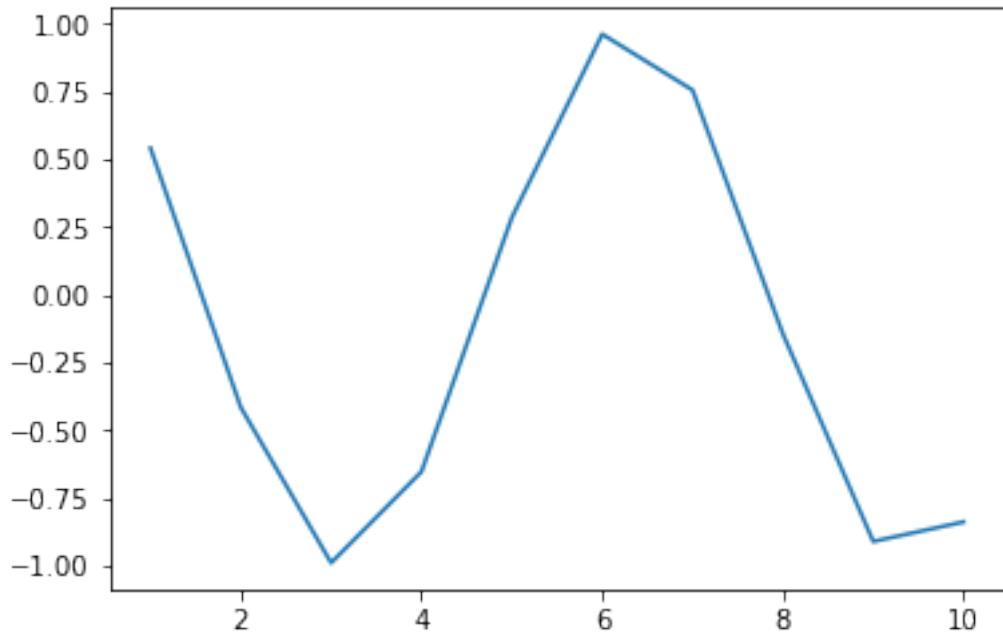
In [19]: A = numpy.array([1,2,3,4,5,6,7,8,9,10])
print(A)
B = numpy.cos(A)
print(B)
plt.plot(A,B)
plt.show()

```

```

[ 1  2  3  4  5  6  7  8  9 10]
[ 0.54030231 -0.41614684 -0.9899925 -0.65364362  0.28366219  0.96017029
 0.75390225 -0.14550003 -0.91113026 -0.83907153]

```



```
In [20]: help(numpy.linspace)
```

```
Help on function linspace in module numpy:
```

```
linspace(start, stop, num=50, endpoint=True, retstep=False, dtype=None, axis=0)
    Return evenly spaced numbers over a specified interval.
```

```
Returns `num` evenly spaced samples, calculated over the
interval [`start`, `stop`].
```

```
The endpoint of the interval can optionally be excluded.
```

```
.. versionchanged:: 1.16.0
    Non-scalar `start` and `stop` are now supported.
```

Parameters

```
start : array_like
```

```
    The starting value of the sequence.
```

```
stop : array_like
```

```
    The end value of the sequence, unless `endpoint` is set to False.
```

```
In that case, the sequence consists of all but the last of ``num + 1``  
evenly spaced samples, so that `stop` is excluded. Note that the step  
size changes when `endpoint` is False.
```

```
num : int, optional
```

```
    Number of samples to generate. Default is 50. Must be non-negative.
```

```
endpoint : bool, optional
    If True, `stop` is the last sample. Otherwise, it is not included.
    Default is True.
retstep : bool, optional
    If True, return (`samples`, `step`), where `step` is the spacing
    between samples.
dtype : dtype, optional
    The type of the output array. If `dtype` is not given, infer the data
    type from the other input arguments.
```

```
.. versionadded:: 1.9.0
```

```
axis : int, optional
    The axis in the result to store the samples. Relevant only if start
    or stop are array-like. By default (0), the samples will be along a
    new axis inserted at the beginning. Use -1 to get an axis at the end.
```

```
.. versionadded:: 1.16.0
```

Returns

```
-----
```

```
samples : ndarray
```

```
    There are `num` equally spaced samples in the closed interval
    ``[start, stop]`` or the half-open interval ``[start, stop)``
    (depending on whether `endpoint` is True or False).
```

```
step : float, optional
```

```
    Only returned if `retstep` is True
```

```
Size of spacing between samples.
```

See Also

```
-----
```

```
arange : Similar to `linspace`, but uses a step size (instead of the
        number of samples).
```

```
geomspace : Similar to `linspace`, but with numbers spaced evenly on a log
            scale (a geometric progression).
```

```
logspace : Similar to `geomspace`, but with the end points specified as
            logarithms.
```

Examples

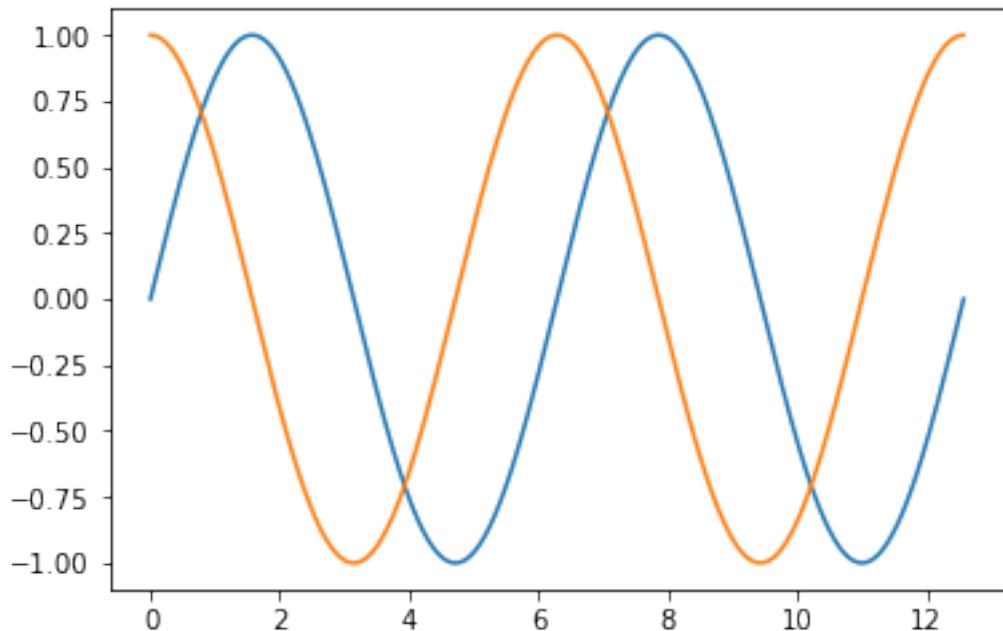
```
-----
```

```
>>> np.linspace(2.0, 3.0, num=5)
array([ 2. ,  2.25,  2.5 ,  2.75,  3. ])
>>> np.linspace(2.0, 3.0, num=5, endpoint=False)
array([ 2. ,  2.2,  2.4,  2.6,  2.8])
>>> np.linspace(2.0, 3.0, num=5, retstep=True)
(array([ 2. ,  2.25,  2.5 ,  2.75,  3. ]), 0.25)
```

Graphical illustration:

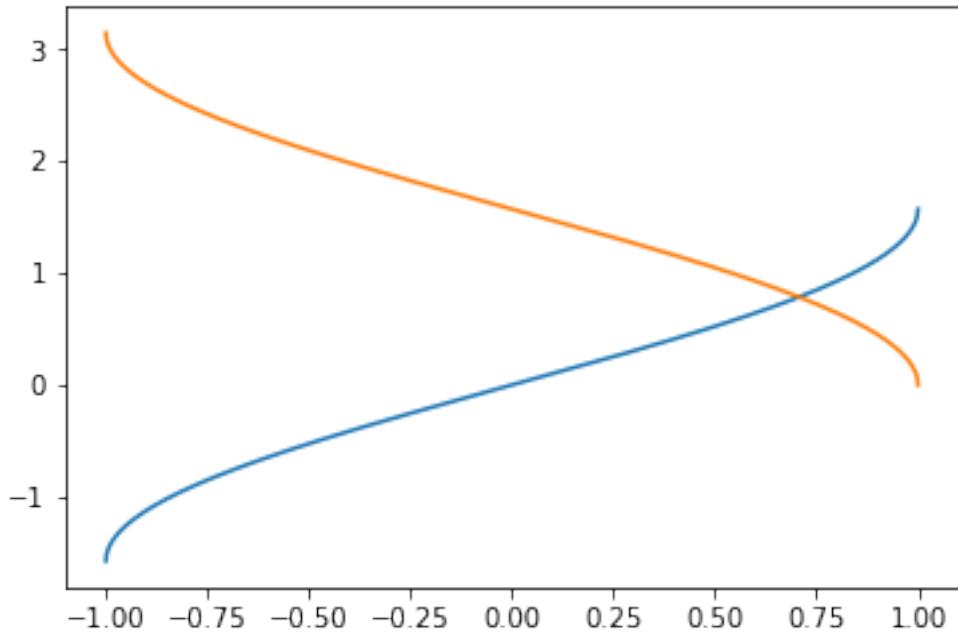
```
>>> import matplotlib.pyplot as plt
>>> N = 8
>>> y = np.zeros(N)
>>> x1 = np.linspace(0, 10, N, endpoint=True)
>>> x2 = np.linspace(0, 10, N, endpoint=False)
>>> plt.plot(x1, y, 'o')
[<matplotlib.lines.Line2D object at 0x...>]
>>> plt.plot(x2, y + 0.5, 'o')
[<matplotlib.lines.Line2D object at 0x...>]
>>> plt.ylim([-0.5, 1])
(-0.5, 1)
>>> plt.show()
```

```
In [21]: L = numpy.linspace(0,4*numpy.pi,1000)
M = numpy.sin(L)
plt.plot(L,M)
N = numpy.cos(L)
plt.plot(L,N)
plt.show()
```



```
In [22]: L = numpy.linspace(-1,1,1000)
M = numpy.arcsin(L)
```

```
plt.plot(L,M)
N = numpy.arccos(L)
plt.plot(L,N)
plt.show()
```



```
In [23]: M = numpy.array([[1,2],[4,5],[7,8]])
print(M)
```

```
[[1 2]
 [4 5]
 [7 8]]
```

```
In [24]: M = numpy.array([[2,1,3,1],[6,3,9,3],[2,1,3,1],[6,3,9,3]])
print(M)
print(M.dot(M))
```

```
[[2 1 3 1]
 [6 3 9 3]
 [2 1 3 1]
 [6 3 9 3]]
[[22 11 33 11]
 [66 33 99 33]
 [22 11 33 11]
 [66 33 99 33]]
```

```
In [25]: M = numpy.array([[npr.randint(1,10) for j in range(3)] for k in range(3)])
          print(M)
          print(M.dot(M))
```

```
[[7 1 3]
 [7 8 2]
 [7 1 5]]
[[ 77  18  38]
 [119  73  47]
 [ 91  20  48]]
```

```
In [26]: n, m = 4,6
          print(numpy.zeros((n,m)))
          print(numpy.ones((m,n)))
          print(numpy.eye(n))
```

```
[[0. 0. 0. 0. 0. 0.]
 [0. 0. 0. 0. 0. 0.]
 [0. 0. 0. 0. 0. 0.]
 [0. 0. 0. 0. 0. 0.]]
[[1. 1. 1. 1.]
 [1. 1. 1. 1.]
 [1. 1. 1. 1.]
 [1. 1. 1. 1.]
 [1. 1. 1. 1.]
 [1. 1. 1. 1.]]
[[1. 0. 0. 0.]
 [0. 1. 0. 0.]
 [0. 0. 1. 0.]
 [0. 0. 0. 1.]]
```

```
In [27]: print(M)
          numpy.shape(M)
```

```
[[7 1 3]
 [7 8 2]
 [7 1 5]]
```

Out[27]: (3, 3)

```
In [28]: def is_vampire(M):
          n,m = numpy.shape(M)
          if not n == m:
              return False
          for i in range(n):
              for j in range(m):
```

```

        if M[i,j] < 0 or M[i,j] > 9:
            return False
    return (M.dot(M) == 11*M).all()

def list_of_matrices(n,m,S):
    L = []
    M = numpy.array([[S[0] for i in range(m)] for j in range(n)])
    L.append(numpy.array(M))
    for i in range(n):
        for j in range(m):
            new_L = []
            for M in L:
                for s in S:
                    N = copy(M)
                    N[i,j] = s
                    new_L.append(N)
            L = list(new_L)
    return L

L = list_of_matrices(2,3,[0,1,2,3,4])
print(len(L))
print(5**6)

def all_vampires(n):
    Vampires = []
    S = [0,1,2,3,4,5,6,7,8,9]
    L = list_of_matrices(n,n,S)
    for M in L:
        if is_vampire(M):
            Vampires.append(M)
    return Vampires

V = all_vampires(2)
print(len(V))
for M in V:
    print(M,M.dot(M))
D = [numpy.linalg.det(M) for M in V]
T = [M.trace() for M in V]
print(D)
print(T)

```

15625
 15625
 25
 [[0 0]
 [0 0]] [[0 0]
 [0 0]]
 [[2 2]

[9 9] [[22 22]
[99 99]]
[[2 3]
[6 9] [[22 33]
[66 99]]
[[2 6]
[3 9] [[22 66]
[33 99]]
[[2 9]
[2 9] [[22 99]
[22 99]]
[[3 3]
[8 8] [[33 33]
[88 88]]
[[3 4]
[6 8] [[33 44]
[66 88]]
[[3 6]
[4 8] [[33 66]
[44 88]]
[[3 8]
[3 8] [[33 88]
[33 88]]
[[4 4]
[7 7] [[44 44]
[77 77]]
[[4 7]
[4 7] [[44 77]
[44 77]]
[[5 5]
[6 6] [[55 55]
[66 66]]
[[5 6]
[5 6] [[55 66]
[55 66]]
[[6 5]
[6 5] [[66 55]
[66 55]]
[[6 6]
[5 5] [[66 66]
[55 55]]
[[7 4]
[7 4] [[77 44]
[77 44]]
[[7 7]
[4 4] [[77 77]
[44 44]]
[[8 3]

```
In [29]: def all_vampires(n,k):
    Vampires = []
    S = list(range(k+1))
    L = list_of_matrices(n,n,S)
    for M in L:
        if is_vampire(M):
            Vampires.append(M)
    return Vampires

print(len(all_vampires(3,4)))
```

7

```
In [36]: from time import time

print(time())

t1 = time()
V = all_vampires(3,4)
t2 = time()
```

```

print(len(V),t2-t1)

L1V = []
Ltime = []
for k in range(9):
    t1 = time()
    V = all_vampires(3,k)
    t2 = time()
    L1V.append(len(V))
    Ltime.append(t2-t1)
    print(k,len(V),t2-t1)

```

```

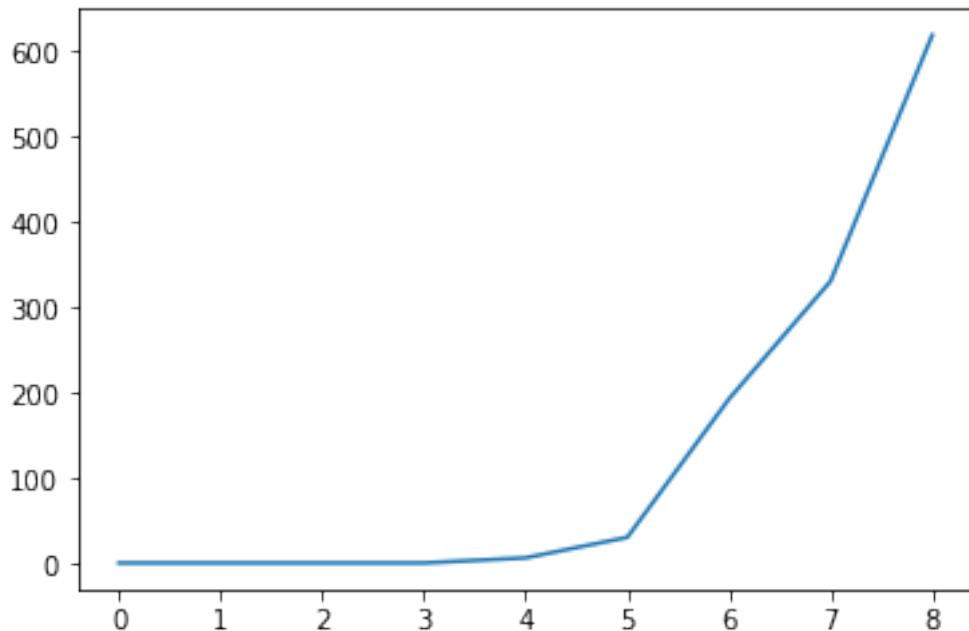
1633895650.8378658
0 1 0.0005712509155273438
1 1 0.020522356033325195
2 1 0.2421119213104248
3 1 3.0042061805725098
4 7 22.243472576141357
5 31 103.05524635314941
6 193 448.24020886421204
7 331 1422.2847096920013
8 619 3996.1434228420258

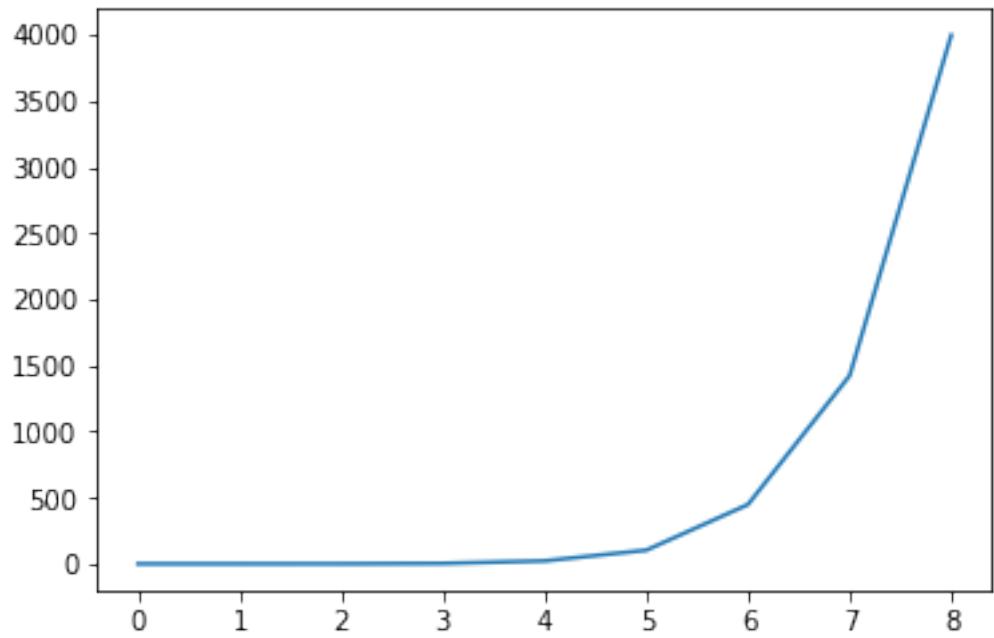
```

```

In [37]: plt.plot(range(len(L1V)),L1V)
plt.show()
plt.plot(range(len(Ltime)),Ltime)
plt.show()

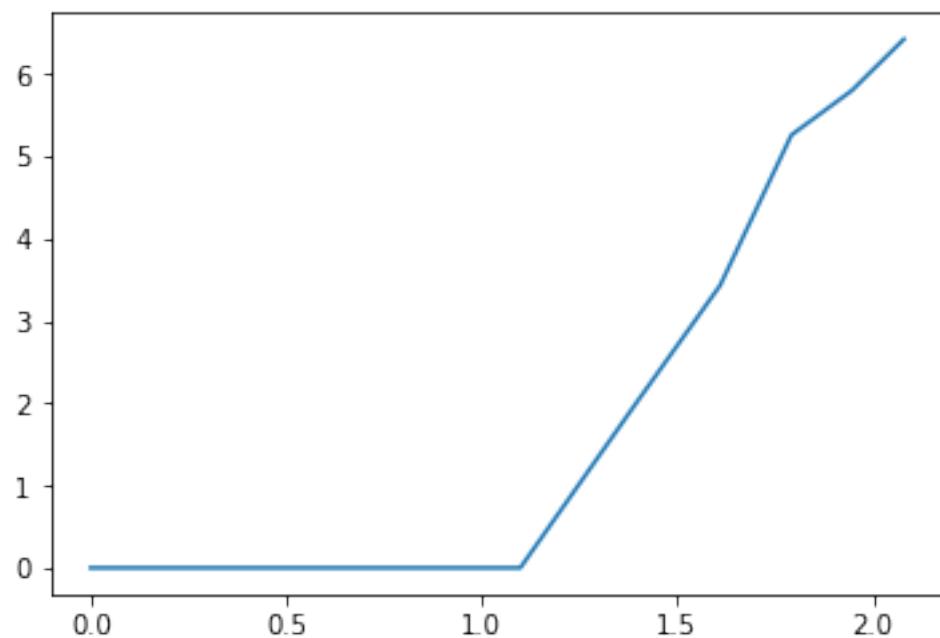
```



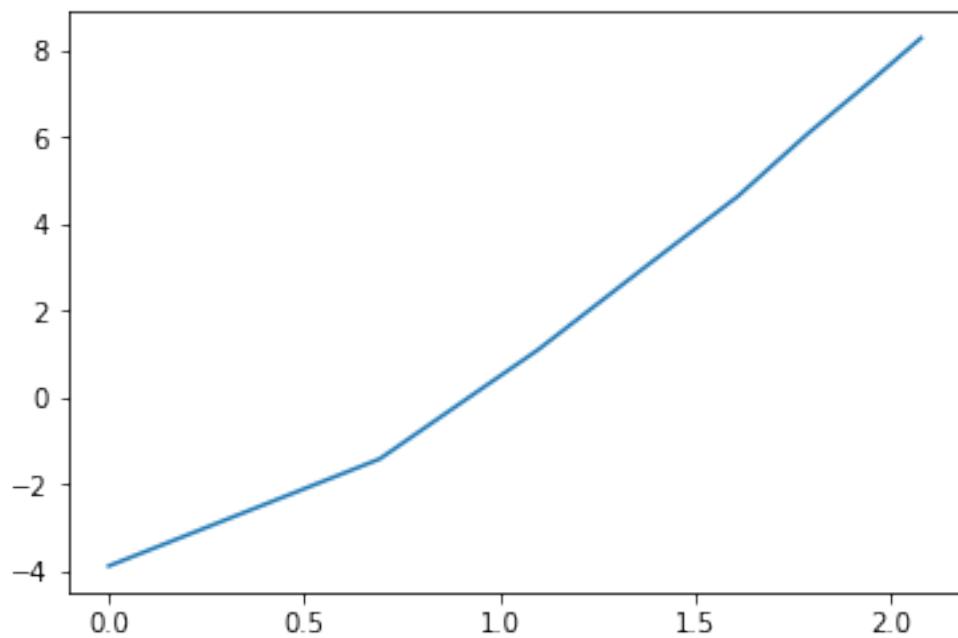


```
In [78]: plt.plot(numpy.log(numpy.array(range(len(L1V)))),numpy.log(numpy.array(L1V)))
plt.show()
plt.plot(numpy.log(numpy.array(range(len(Ltime)))),numpy.log(numpy.array(Ltime)))
plt.show()

/usr/local/lib/python3.6/dist-packages/ipykernel_launcher.py:1: RuntimeWarning: divide by zero
"""\Entry point for launching an IPython kernel.
```



```
/usr/local/lib/python3.6/dist-packages/ipykernel_launcher.py:3: RuntimeWarning: divide by zero  
  This is separate from the ipykernel package so we can avoid doing imports until
```



```

In [77]: D = {k : [(i,j) for i in range(1,7) for j in range(1,7) if abs(i-j) == k] for k in range(8)}
          print(D)

def loi_de_la_difference(n):
    D = {k : 0 for k in range(n)}
    for i in range(1,n+1):
        for j in range(1,n+1):
            D[abs(i-j)] += 1
    return D

def somme(L):
    s = 0
    for a in L.values():
        s += a
    return s

def esperance(L):
    e = 0
    for x in L.keys():
        e += x*L[x]
    return e/somme(L)

def variance(L):
    e = 0
    for x in L.keys():
        e += (x**2)*L[x]
    return e/somme(L) - esperance(L)**2

def mode(L):
    y, m = 0, 0
    for x in L.keys():
        if L[x] >= m:
            y = copy(x)
            m = L[x]
    return y

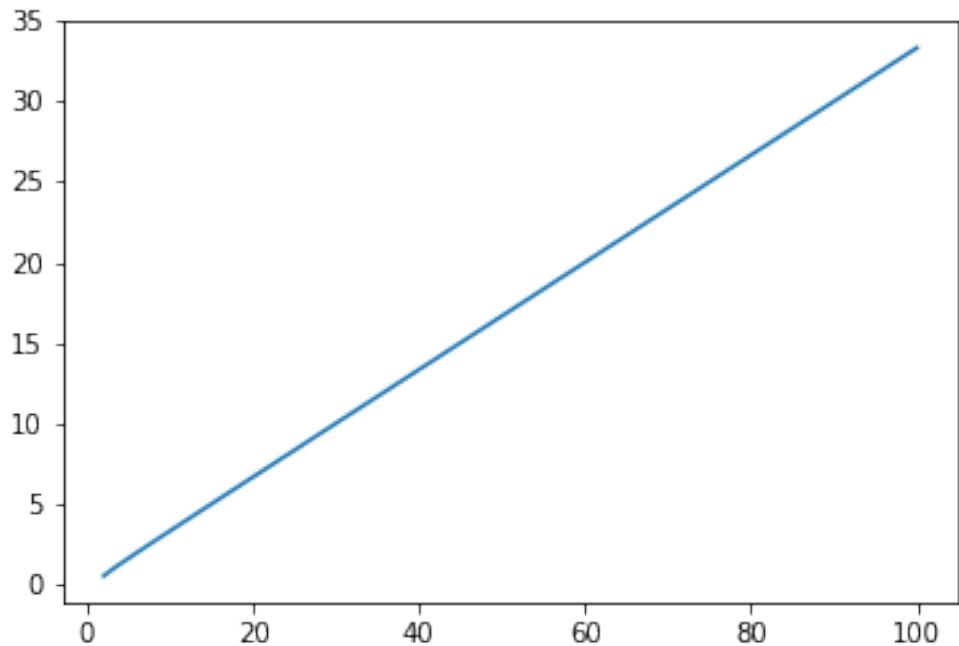
N = list(range(2,101))
E, V, M = [], [], []
for n in N:
    L = loi_de_la_difference(n)
    E.append(esperance(L))
    V.append(variance(L))
    M.append(mode(L))

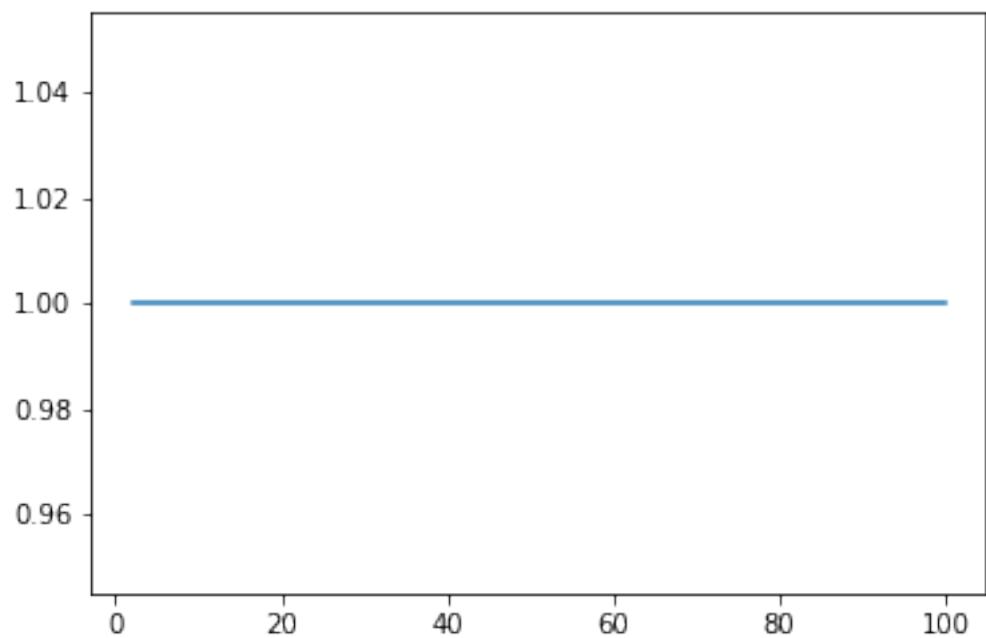
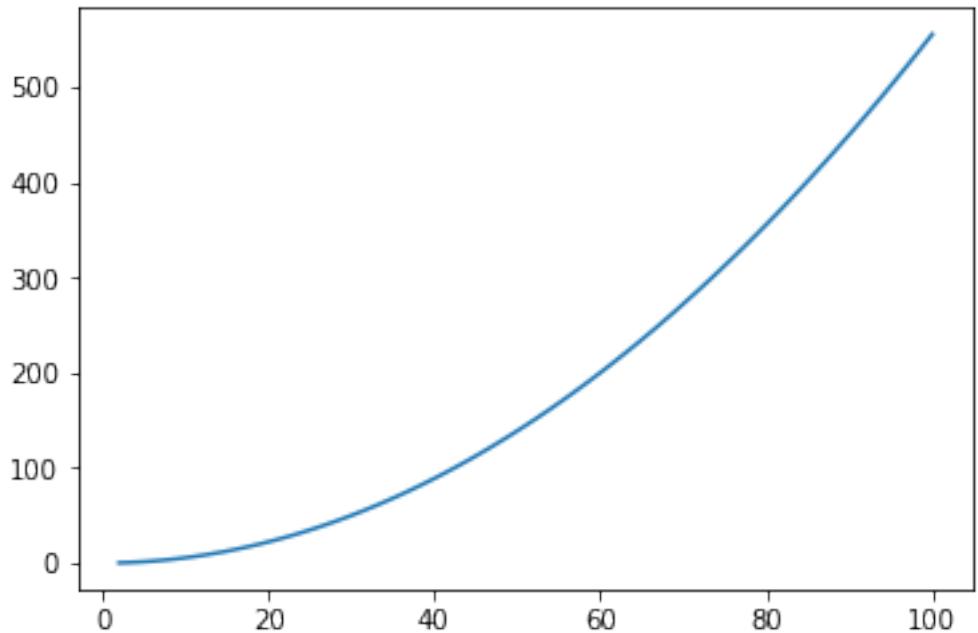
plt.plot(N,E)
plt.show()
plt.plot(N,V)
plt.show()

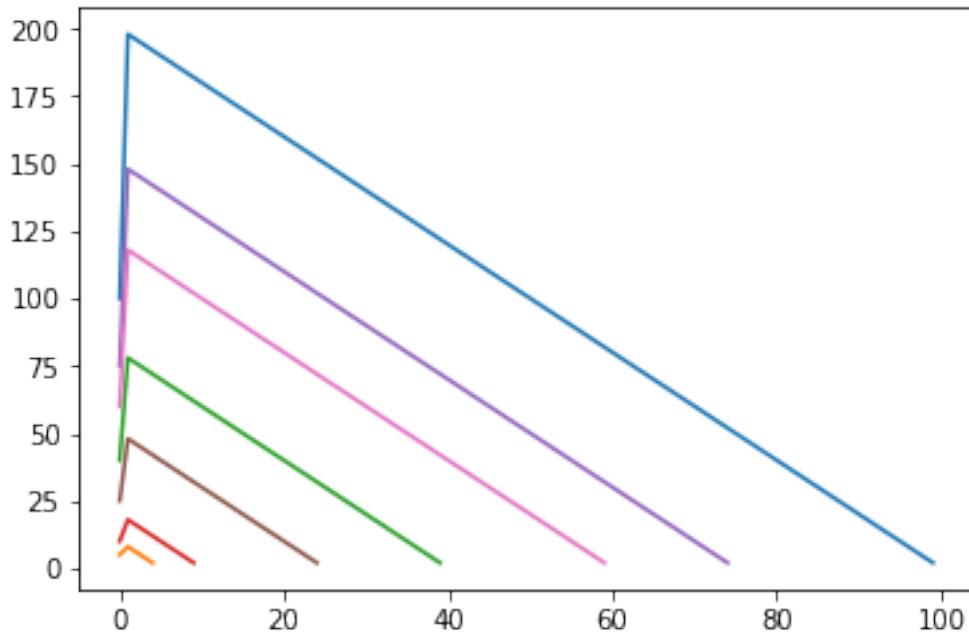
```

```
plt.plot(N,M)
plt.show()
for n in {5,10,25,40,60,75,100}:
    L = loi_de_la_difference(n)
    plt.plot(L.keys(),L.values())
plt.show()
```

```
{0: [(1, 1), (2, 2), (3, 3), (4, 4), (5, 5), (6, 6)], 1: [(1, 2), (2, 1), (2, 3), (3, 2), (3, 4), (4, 1), (4, 3), (5, 1), (5, 2), (5, 3), (6, 1), (6, 2), (6, 3), (6, 4)], 2: [(1, 3), (2, 2), (3, 1), (3, 4), (4, 1), (4, 2), (4, 3), (5, 1), (5, 2), (5, 3), (5, 4), (6, 1), (6, 2), (6, 3), (6, 4), (6, 5)], 3: [(1, 4), (2, 3), (3, 2), (4, 1), (4, 2), (4, 3), (5, 1), (5, 2), (5, 3), (5, 4), (6, 1), (6, 2), (6, 3), (6, 4), (6, 5), (6, 6)], 4: [(1, 5), (2, 4), (3, 3), (4, 2), (5, 1), (5, 2), (5, 3), (5, 4), (6, 1), (6, 2), (6, 3), (6, 4), (6, 5), (6, 6), (6, 7)], 5: [(1, 6), (2, 5), (3, 4), (4, 3), (5, 2), (6, 1), (6, 2), (6, 3), (6, 4), (6, 5), (6, 6), (6, 7), (6, 8)], 6: [(1, 7), (2, 6), (3, 5), (4, 4), (5, 3), (6, 2), (6, 3), (6, 4), (6, 5), (6, 6), (6, 7), (6, 8), (6, 9)], 7: [(1, 8), (2, 7), (3, 6), (4, 5), (5, 4), (6, 3), (6, 4), (6, 5), (6, 6), (6, 7), (6, 8), (6, 9), (6, 10)], 8: [(1, 9), (2, 8), (3, 7), (4, 6), (5, 5), (6, 4), (6, 5), (6, 6), (6, 7), (6, 8), (6, 9), (6, 10), (6, 11)], 9: [(1, 10), (2, 9), (3, 8), (4, 7), (5, 6), (6, 5), (6, 6), (6, 7), (6, 8), (6, 9), (6, 10), (6, 11), (6, 12)], 10: [(1, 11), (2, 10), (3, 9), (4, 8), (5, 7), (6, 6), (6, 7), (6, 8), (6, 9), (6, 10), (6, 11), (6, 12), (6, 13)]}
```







```
In [75]: N = 35
Peri = []
Aire = []
for a in range(1,N+1):
    for b in range(1,N+1):
        for c in range(1,N+1):
            if 2*max(a,b,c) < a+b+c:
                Peri.append(a+b+c)
                s = (a+b+c)/2
                Aire.append(sqrt(s*(s-a)*(s-b)*(s-c)))

plt.plot(Peri,Aire,'b')
plt.plot()
L = linspace(3,3*N,500)
plt.plot(L, numpy.sqrt(3)/36 * L**2)
plt.plot(L, 1/4*numpy.sqrt((L-1)**2-1))
plt.show()
```

